Ray Tracing: BRDF & Sampling

CSCI 4239/5239
Advanced Computer Graphics
Spring 2025

Theoretical foundations
Ray Tracing from the Ground Up Chapters 13-15

- Bidirectional Reflectance Distribution Function
  - BRDF
  - Describes how light is reflected on each bounce
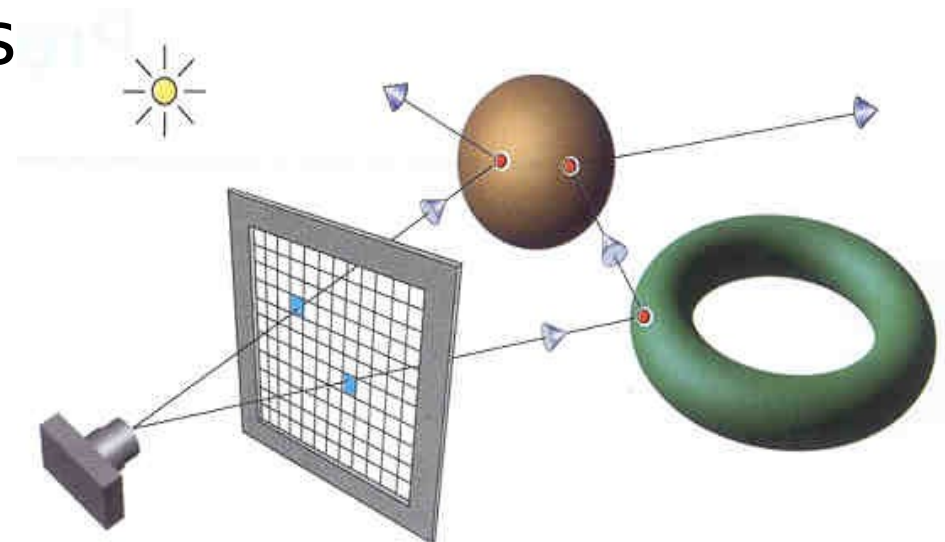 - Chains to transfer colors

Figure 1. The ray-tracing process.

Radiometric Quantities

- Radiant Energy Q (J)
- Radiant Flux $\phi = dQ/dt$ (W)
- Radiant Flux Density $d\phi/dA$ (W/m^2)
- Irradiance E [Arriving flux density]
- Radiant exitance M [Leaving flux density]
- Radiant Intensity I $d\phi/d\omega$ (W/m^2/sr)
- Radiance L $d^2\phi/dAd\omega$ (W/m^2/sr)

Ray Properties

- Radiance is constant along rays
- Radiance can be defined at the eye
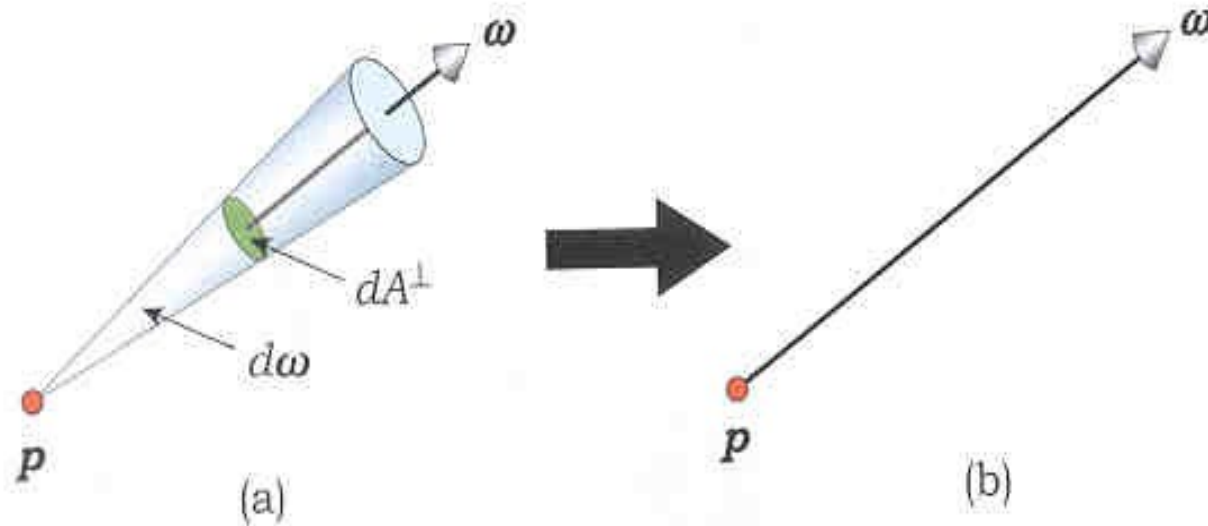- Radiance can be defined at a point

Figure 13.1. (a) Radiant flux in a cone of incident angles $d\omega$ passing through a surface element $dA^{\perp}$. (b) In the limit $d\omega \to 0$ and $dA \to 0$, the radiance is defined as coming from a single direction ω. The point p can be an arbitrary point in space.

Angular Dependence on Irradiance

- Lambert's Law
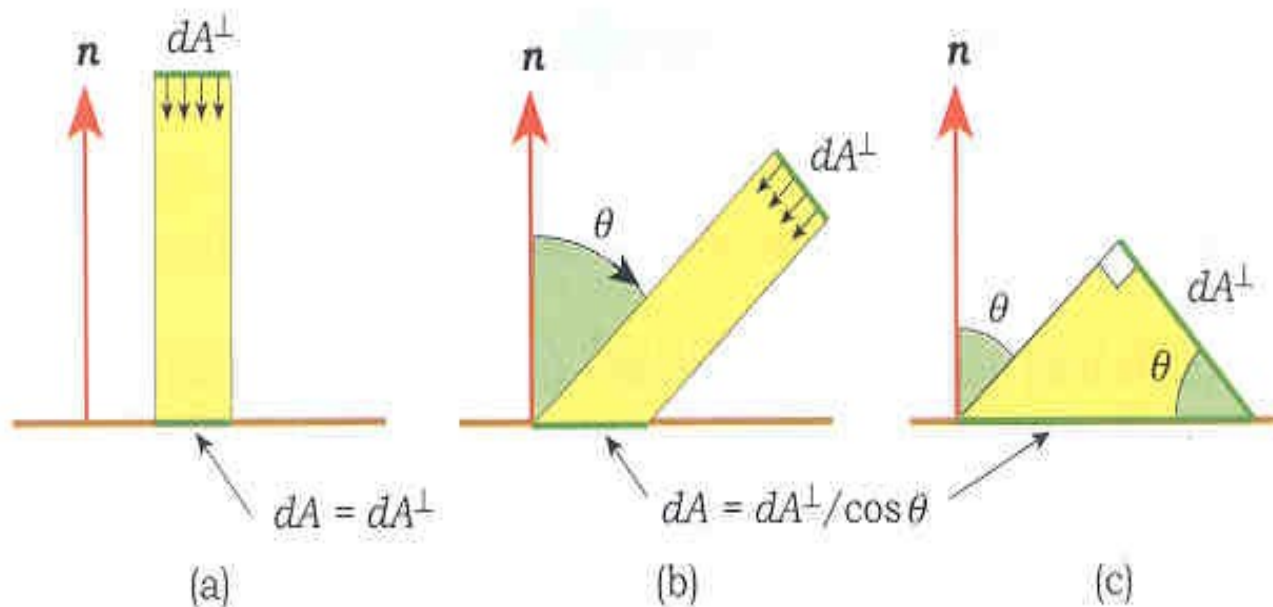 - $L = d^2\phi/dA \cdot d\omega \cdot \cos\theta$

Figure 13.2. (a) and (b) Irradiance spreads out over a larger area as the incidence angle θ increases. (c) An enlarged view of the incident beam.
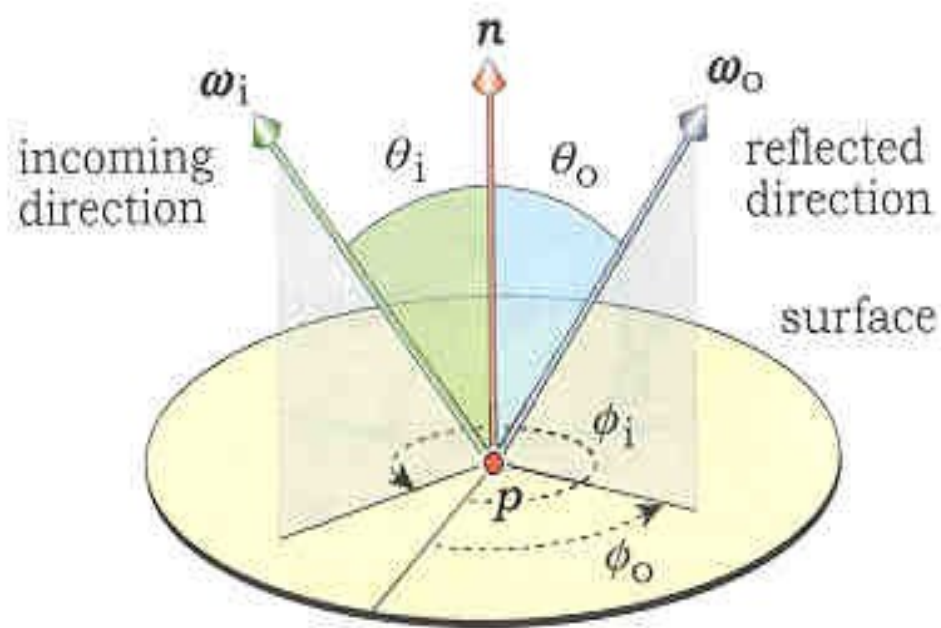
Notation and Directions

Figure 13.3. The incoming direction ω_i and reflected direction ω_o point away from the surface and are on the same side of the surface as the normal. Each direction is defined by its polar and azimuth angles (θ, ϕ). These are arbitrary directions; for perfect mirror reflection, $\phi_o = \phi_i \pm \pi$, as illustrated in Figure 24.2(b).

BRDF

- Definition
  - $f(p, \omega_i, \omega_o) = dL_o(p, \omega) \, / \, dL_i(p, \omega) \cos \theta_i \, d\omega_i$
- Properties
  - Reciprocity
    - $f(p, \omega_i, \omega_o) = f(p, \omega_o, \omega_i)$
  - Linearity
    - Sum all BRDFs at a point
  - Conservation of energy
    - Total re-radiated energy must be less than incident

Common BRDFs
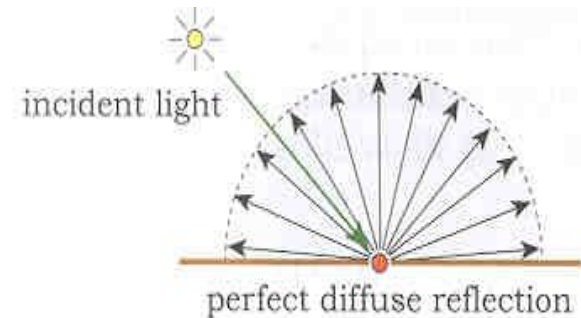
- Diffuse $f(p, \omega_i, \omega_o) = M_d(p)$

incident light

perfect diffuse reflection

Figure 13.6. Light being scattered from a perfectly diffuse surface.

- Specular $f(p, \omega_i, \omega_o) = M_s(p) \, (R \cdot \omega_o)^S$
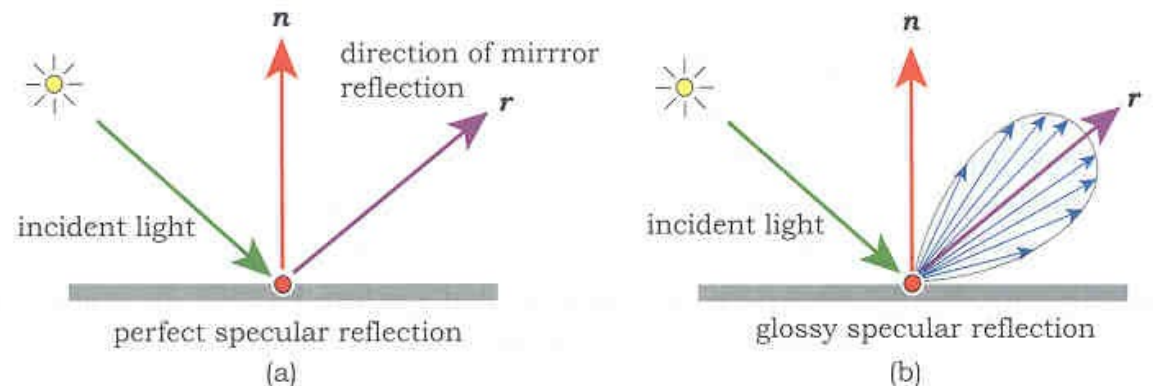 - $R = 2(N \cdot L)N - L$

n

direction of mirrror reflection
r

incident light

perfect specular reflection

(a)

n

r

incident light

glossy specular reflection

(b)

Figure 14.3. (a) Perfect specular reflection; (b) glossy specular reflection.
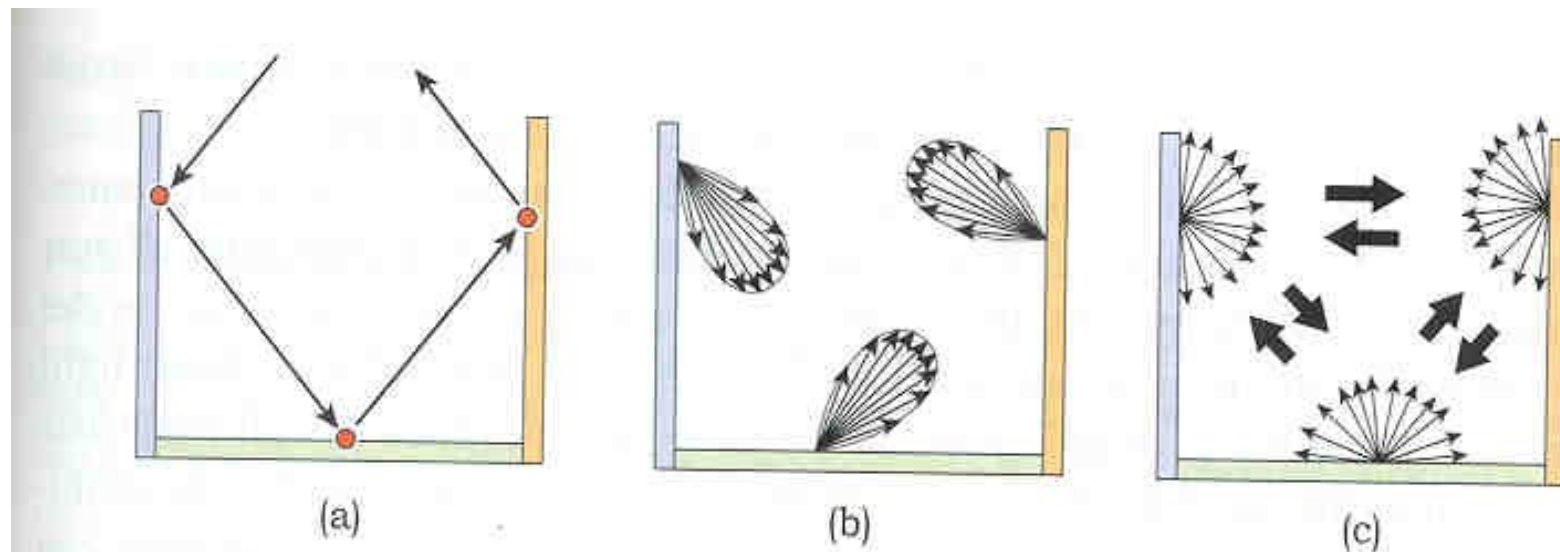
Bouncing Rays from Surfaces

Figure 14.4. (a) Mirror reflection can be modeled by tracing a single reflected ray at each hit point; (b) modeling glossy specular light transport between surfaces requires many rays to be traced per pixel; (c) modeling perfect diffuse light transport between surfaces also requires many rays to be traced per pixel.
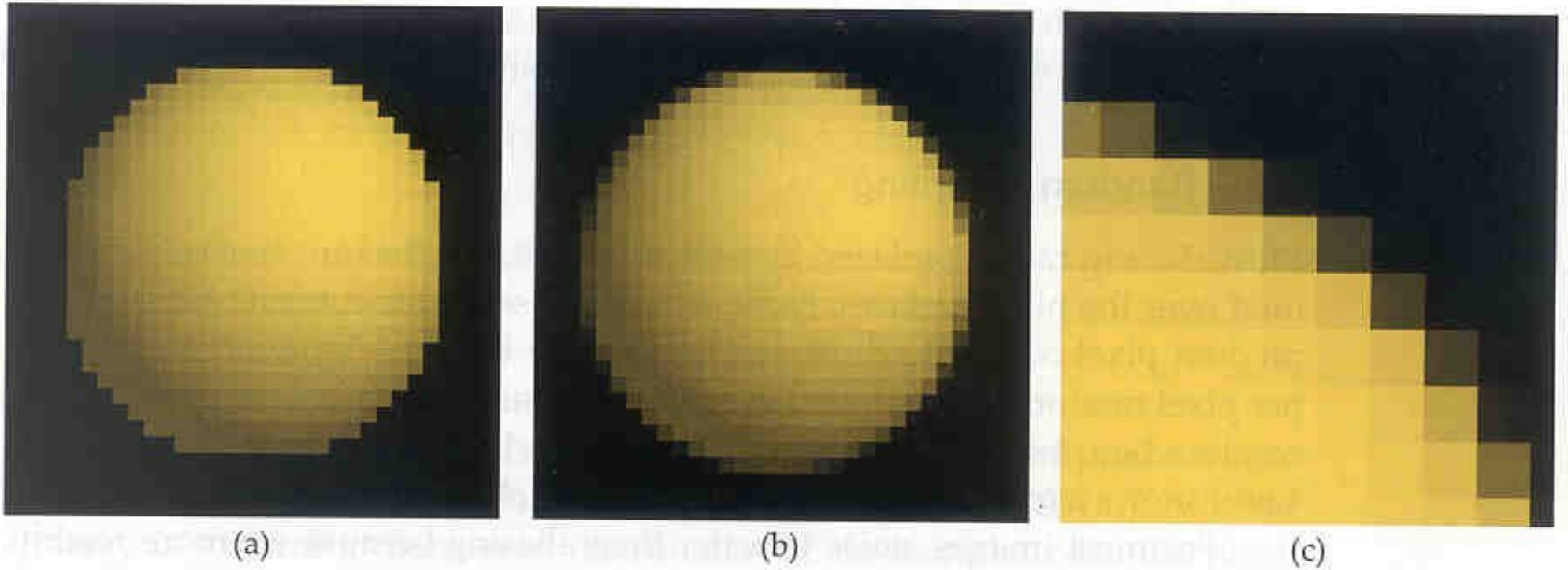
Antialiasing

Figure 4.4. Shaded sphere: (a) one sample per pixel; (b) 16 samples per pixel; (c) enlarged view of top-right section of (b).
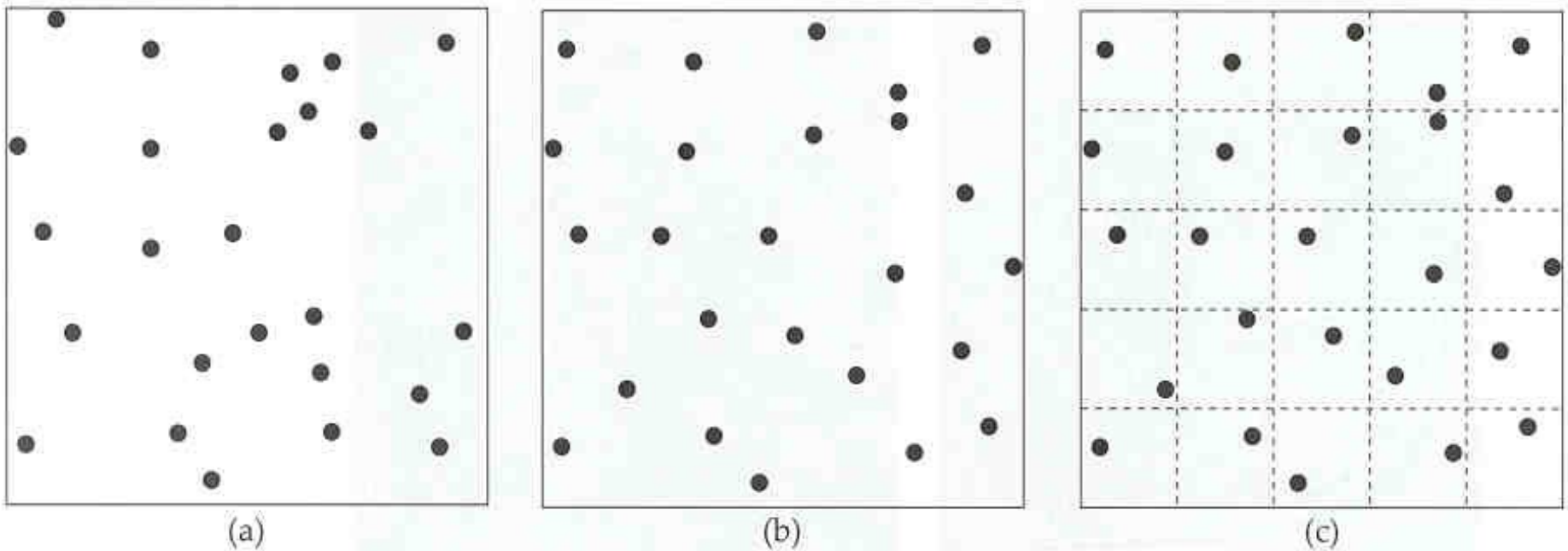
Super-sampling Pixels

Figure 4.7. (a) 25 random samples in a pixel; (b) 25 jittered samples; (c) same as (b) but with sub-grid lines shown.
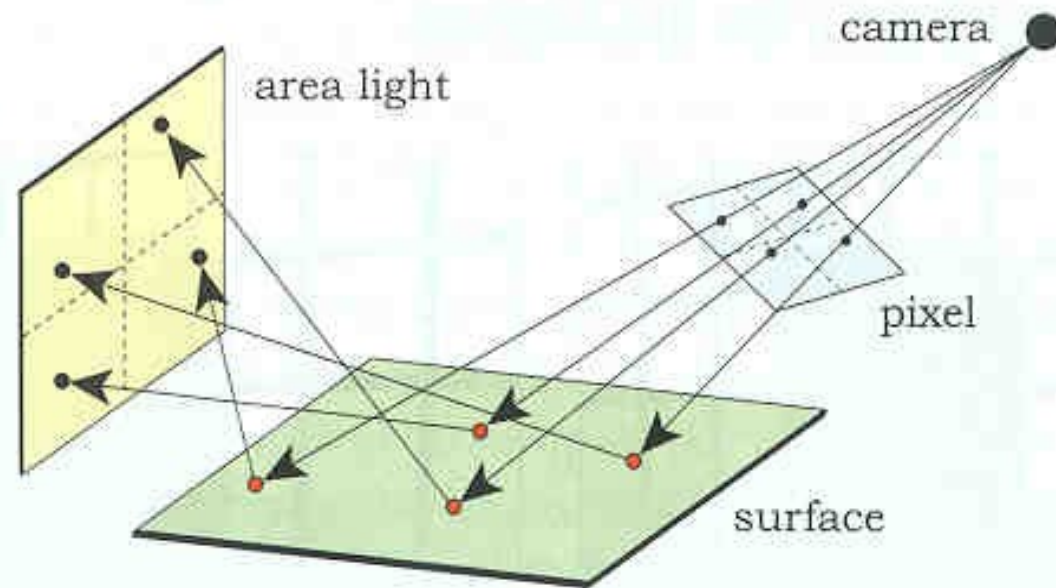
Super-sampling Area Lights

Figure 5.2. Shading a surface with an area light and four samples per pixel.
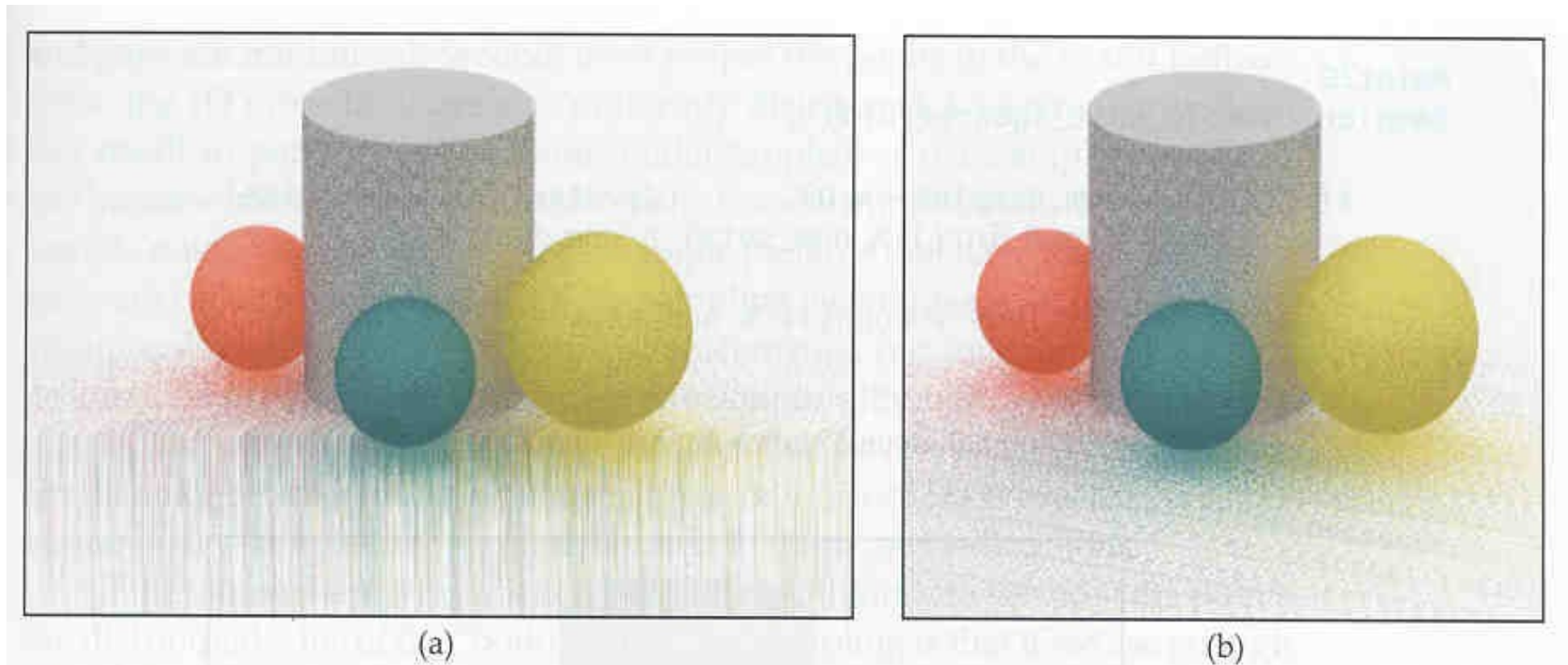
Side-effects of Sampling Pattern

Figure 5.6. Global illumination images that exhibit bad aliasing caused by using the same samples in vertical columns (a) and in a regular horizontal displacement (b).

Depth of Field

- Important for realism
  - Background is "fuzzy"
- Partly out of focus
  - Imperfect optics
  - Turbulence
- Graphic backgrounds are often too perfect

Image courtesy of Tania Humphreys
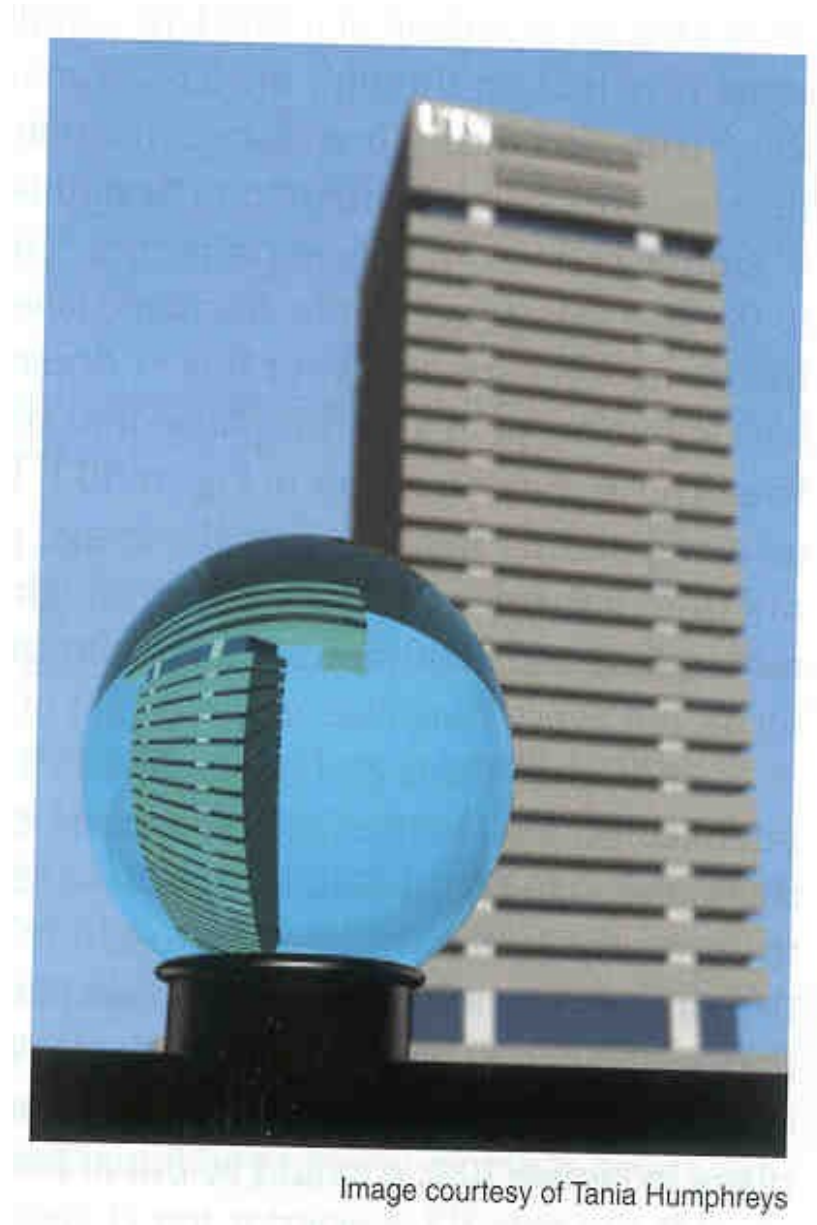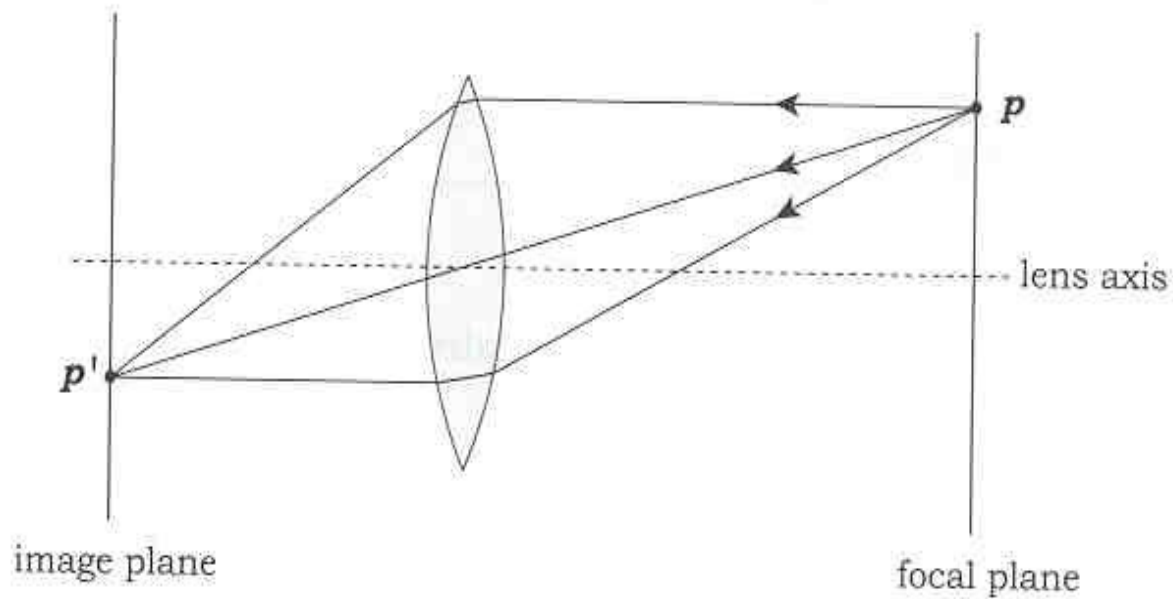
Thin Lens Theory

Figure 10.1. Cross section through a thin lens showing a focal plane and its corresponding image plane.
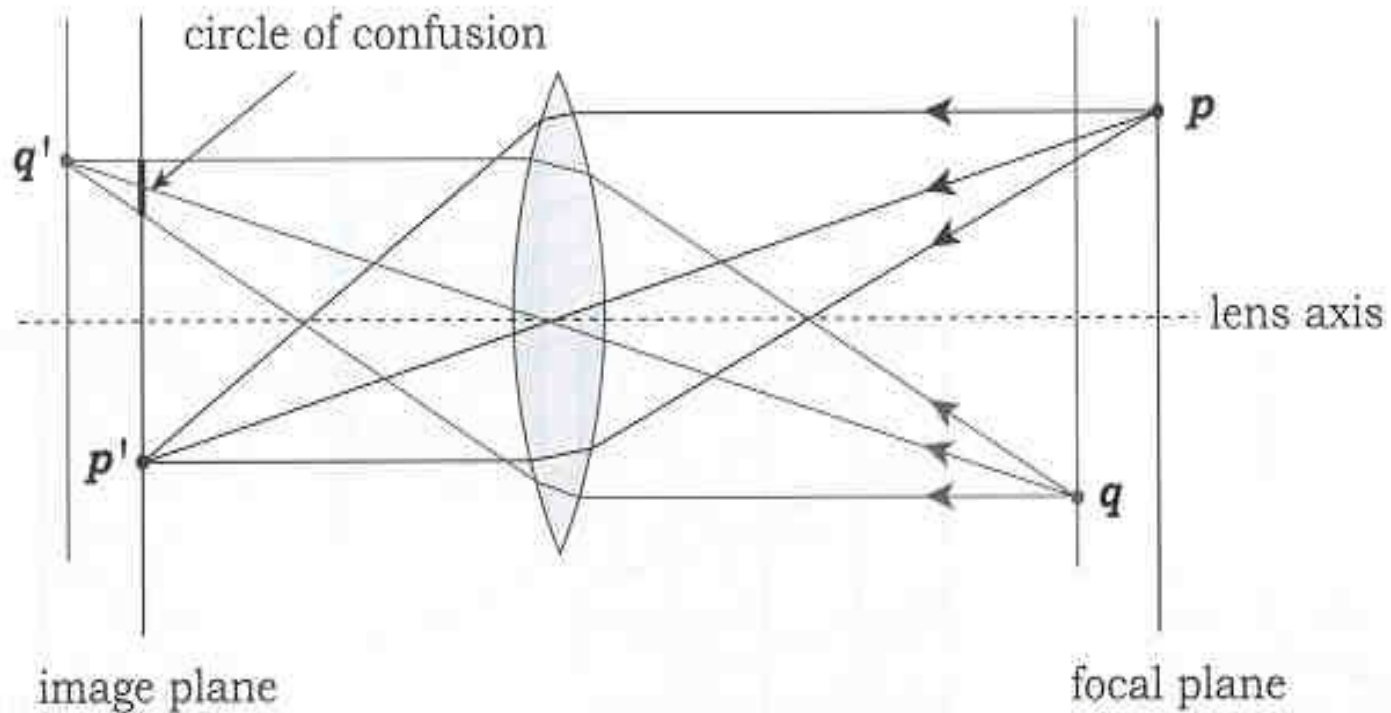
Out of Focus Images

Figure 10.2. Rays starting a point q go through the image plane of p at different locations, with the result that q will appear out of focus.
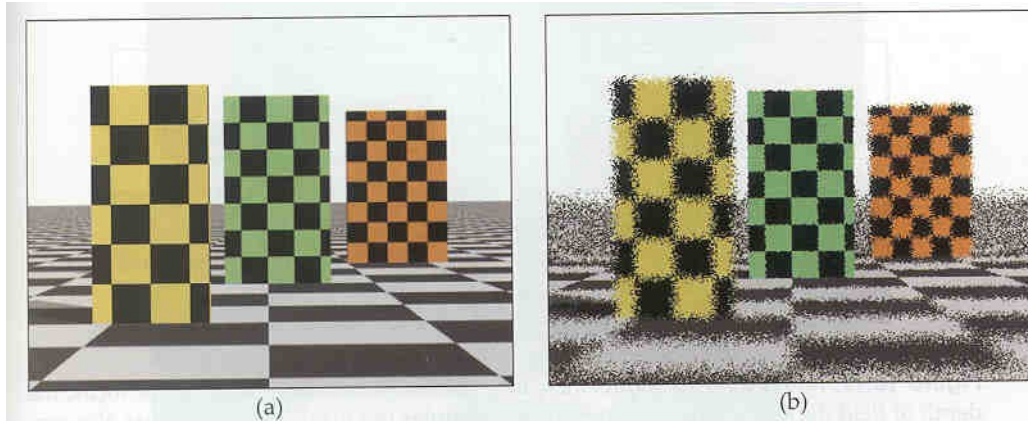
Depth of Field Results

Figure 10.9. (a) When the lens radius is zero, the image is the same as a pinhole-camera image with everything in focus; (b) noisy image from using one random sample per pixel.
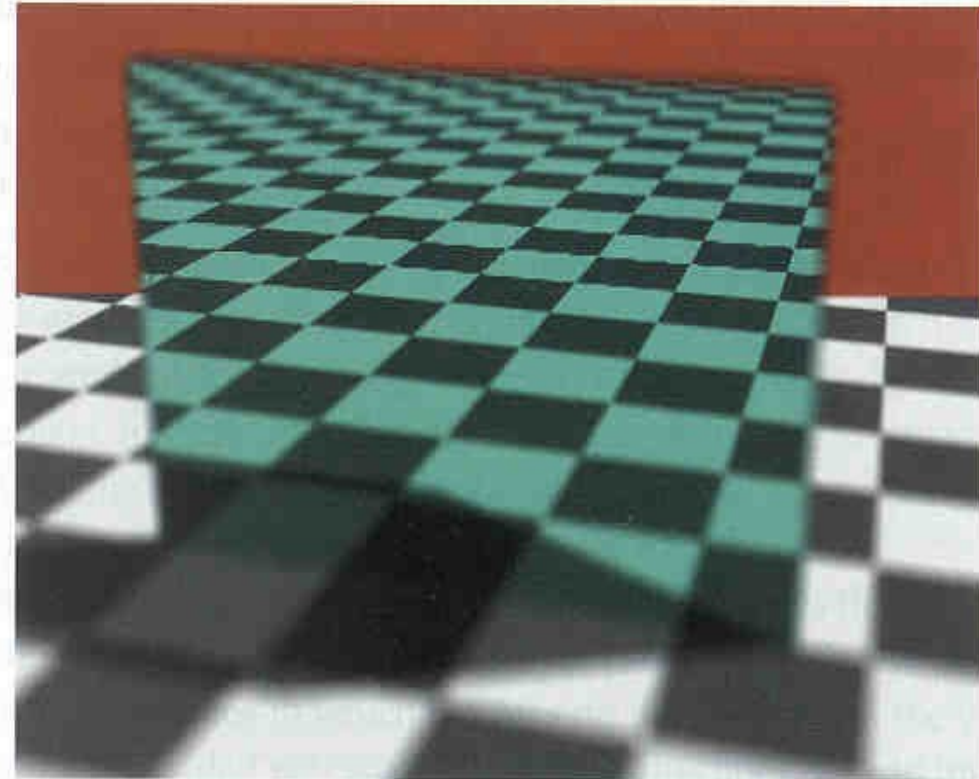

Figure 10.12. Mirrored surface.

Ambient Occlusion

- Floor has a vague shadow outline

- Parts of object near floor is darker

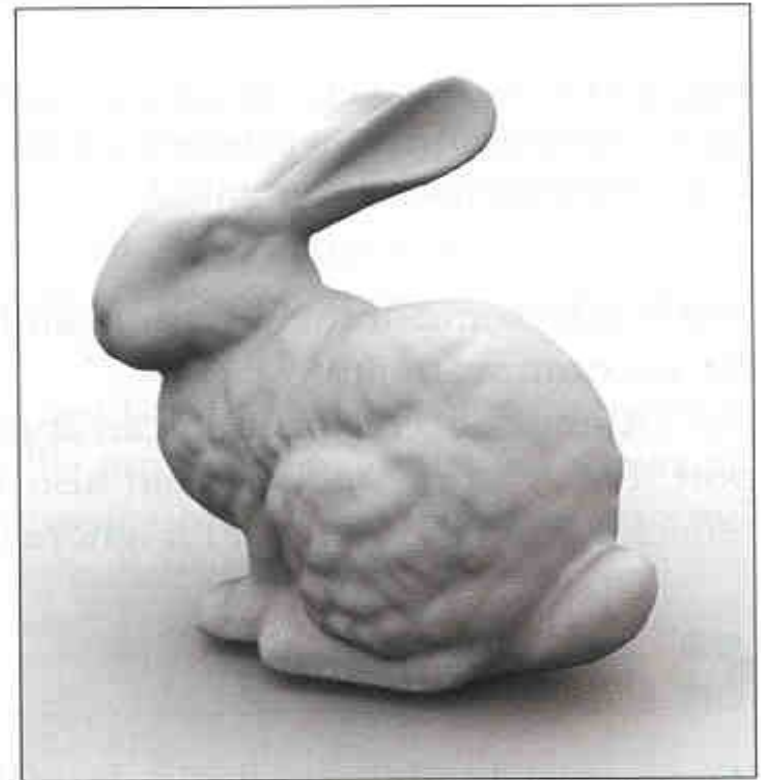- Ambient light is not anisotropic and uniform

Image courtesy of Mark Howard, Stanford bunny model courtesy of Greg Turk and the Stanford University Graphics Laboratory
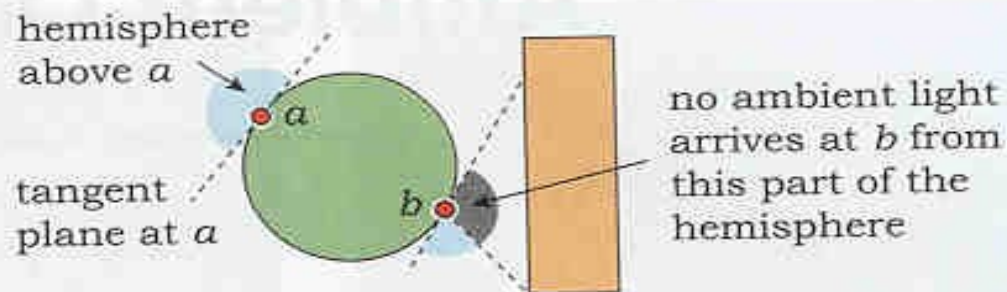
Computing Ambient Occlusion

hemisphere above a

tangent plane at a

a

b

no ambient light arrives at b from this part of the hemisphere

Figure 17.1. Point a on the sphere receives the maximum amount of ambient light because the box isn't visible; point b doesn't receive the maximum amount because the box blocks some of the incoming ambient light.
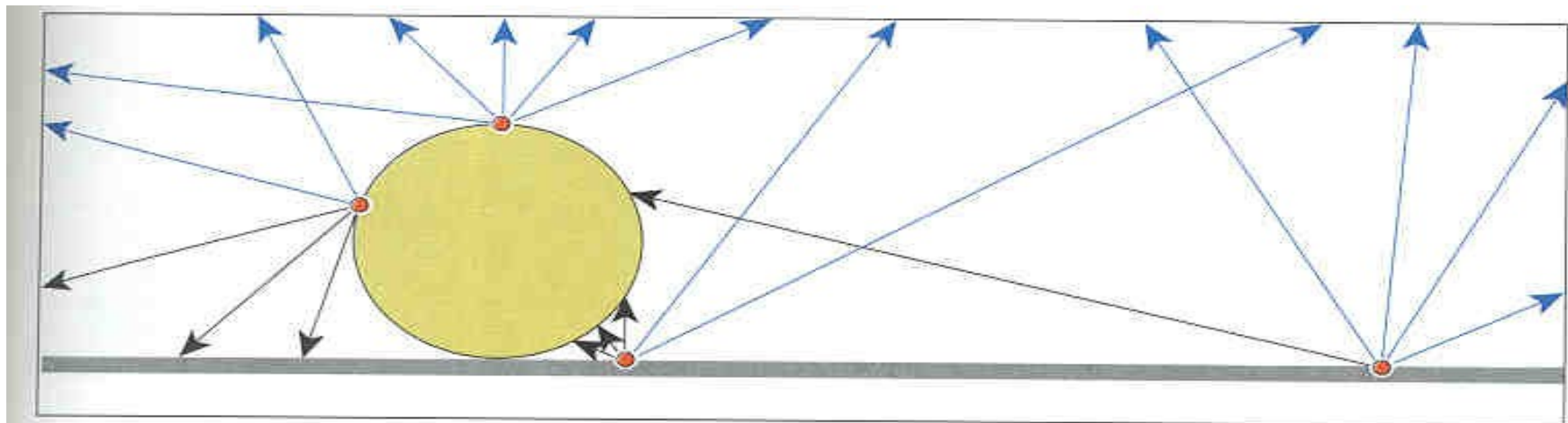
Figure 17.2. Various hit points on the plane and the sphere, with sample shadow rays.

Ambient Occlusion Results
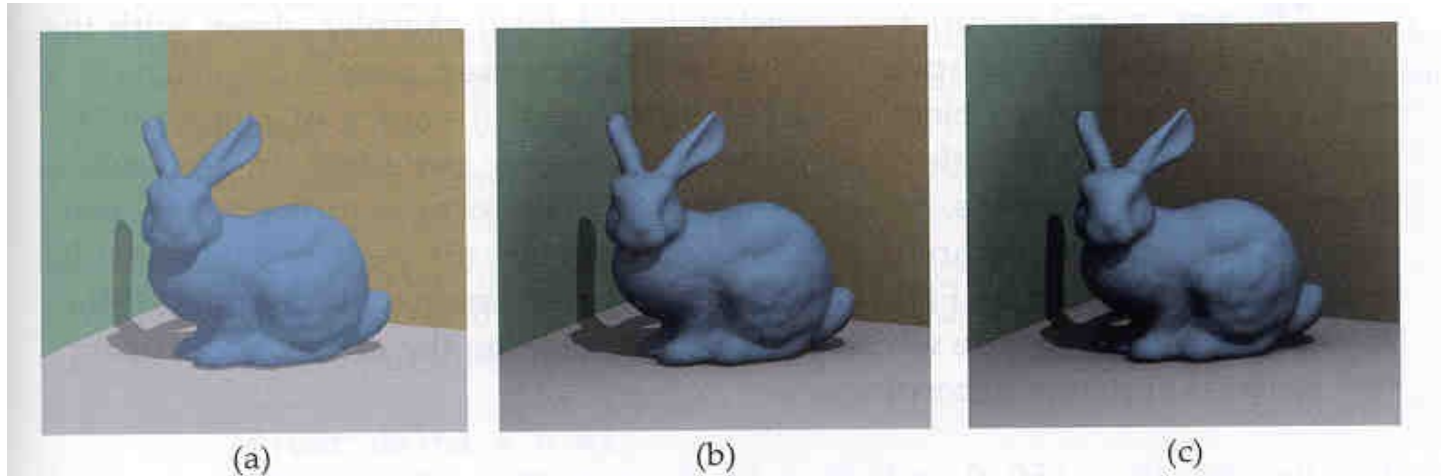

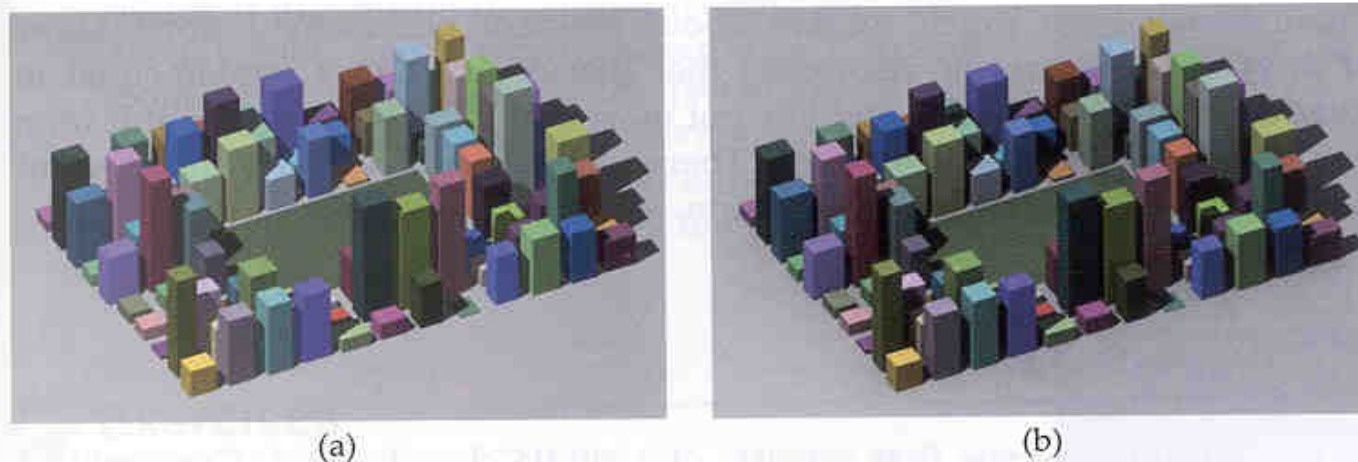Figure 17.12. Bunny scene rendered with 256 samples per pixel: (a) min_amount = 1; (b) min_amount = 0.25; (c) min_amount = 0.

Figure 17.13. Random boxes rendered with 64 samples per pixel: (a) min_amount = 1.0; (b) min_amount = 0.25.

Mirror Reflection

- Mirror reflections are a signature of ray tracing
    - Shiny objects
        - Glass
        - Metal
    - Multiple reflections may occur
- Occurs naturally in ray tracing
- Requires tracing ray through multiple bounces
- Adds significant effort

Reflective spheres by Burt Flugleman, Rundle Street Mall, Adelaide. Photograph by Kevin Suffern.

Conservation of Energy

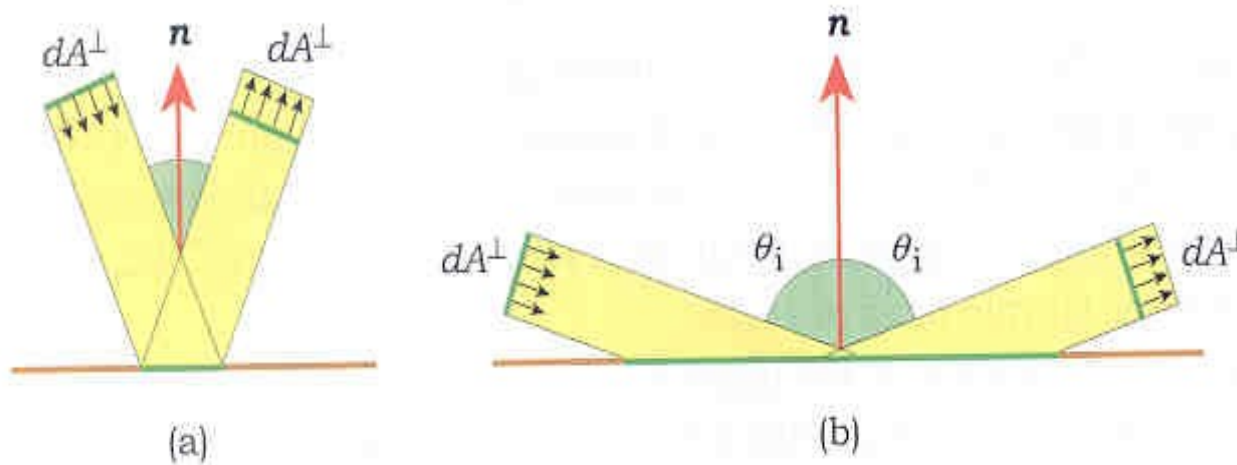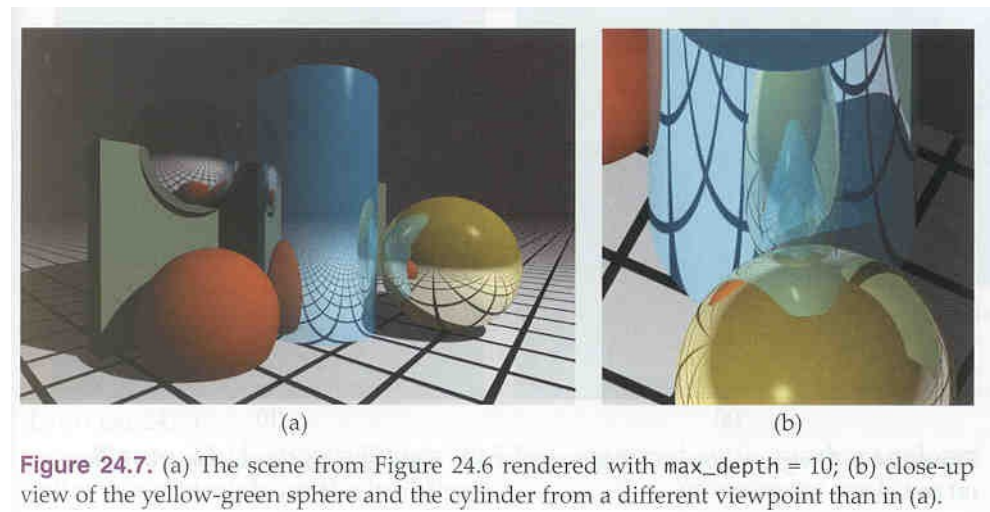- Mirrors reflect almost all the energy
- Retains beam geometry

Figure 24.3. When a beam of light is reflected from a perfect mirror, its cross section area is unchanged after reflection and is therefore independent of the angle of incidence θ_i.

Number of Reflections

- 0 dull
- 1 "simple" mirror
- >1 "hall of mirrors"
- Effort grows with number of bounces

Figure 24.6. A scene rendered with max_depth = 0 (a) and max_depth = 1 (b).

Figure 24.7. (a) The scene from Figure 24.6 rendered with max_depth = 10; (b) close-up view of the yellow-green sphere and the cylinder from a different viewpoint than in (a).
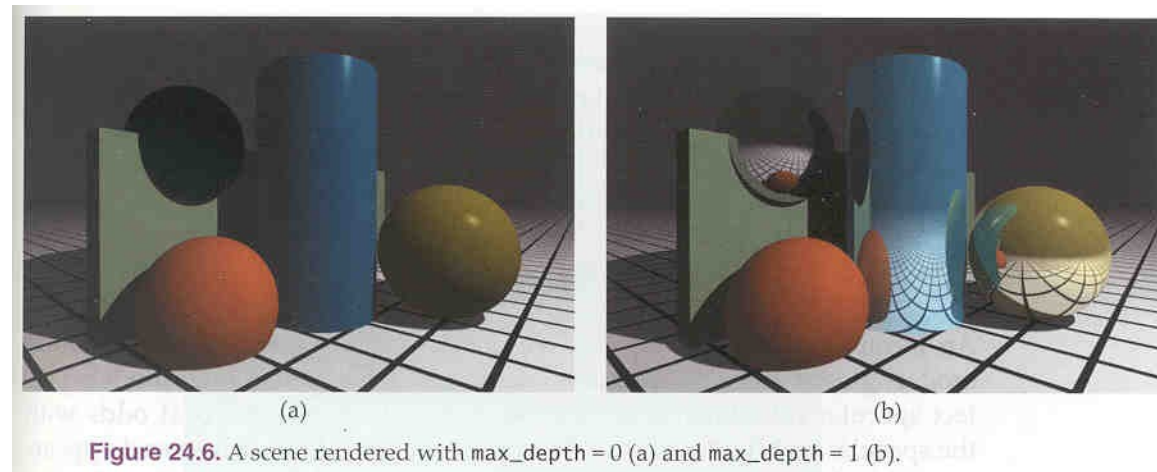
Hall of Mirrors
(Showcases Ray Tracing)

(a)   (b)
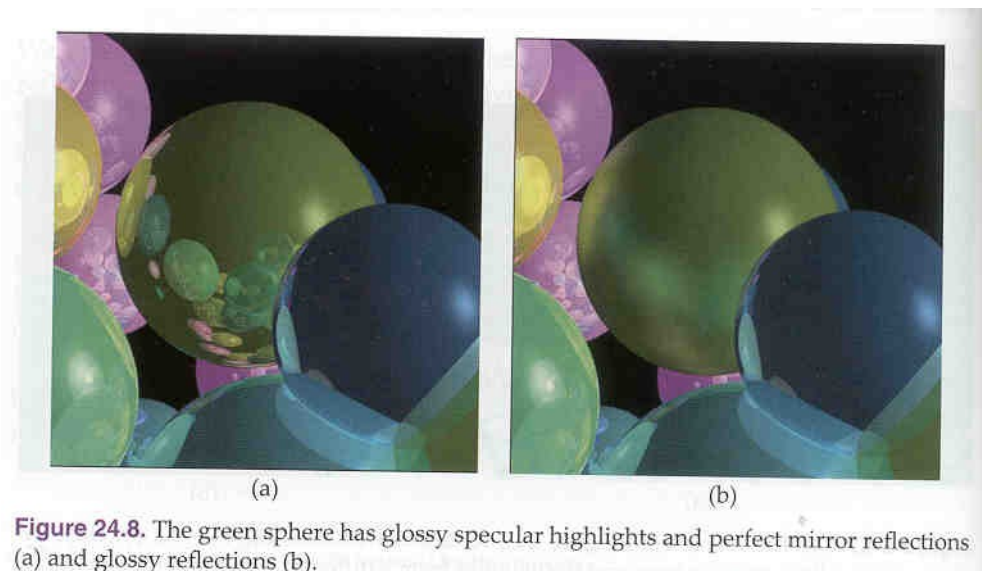
Figure 24.18. (a) Hall of mirrors with max_depth = 19; (b) close-up view of the multiple reflections between the floor mirror and the sphere.

Mirror vs Glossy Reflection

- Mirror reflections are "perfect"
- Glossy reflections are "imperfect"
  - Reflected ray = $2(N \cdot V)N - V + \varepsilon$
  - Super-sample for many values of ε

Figure 24.8. The green sphere has glossy specular highlights and perfect mirror reflections (a) and glossy reflections (b).
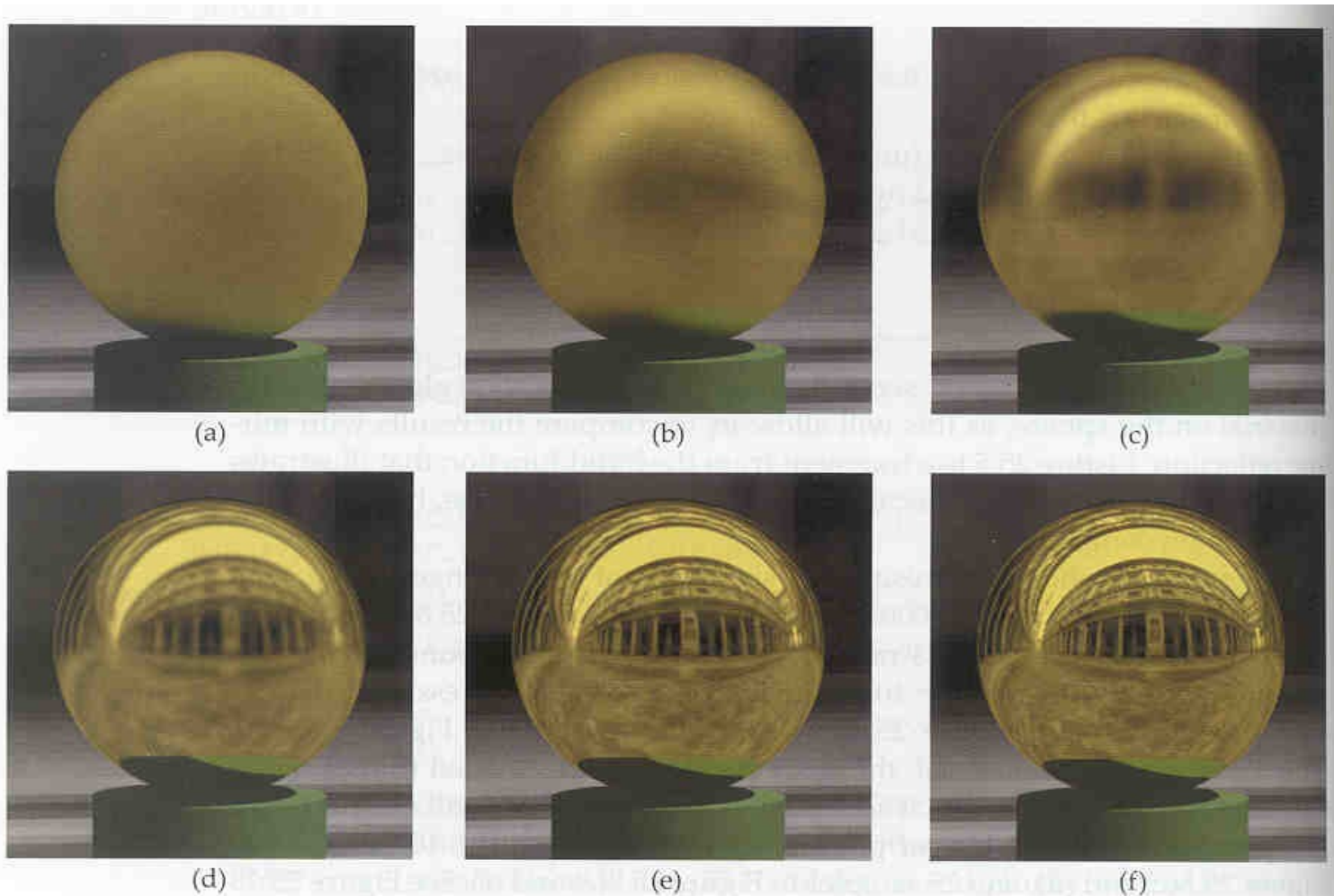
Degrees of perfection

Figure 25.8. Glossy sphere surrounded by the Uffizi image and rendered with the following values of e: (a) 1.0; (b) 10.0; (c) 100.0; (d) 1000.0; (e) 10000.0; (f) 100000.0.

Reflections in Raster Methods

- Two possible approaches
  - Textures (image space)
  - Virtual objects (object space)
- Both approaches requires rendering the scene multiple times
- Mirrors can be planar or curved
- Mirrors are "windows" to the virtual scene

Virtual Objects

- Draw object where they seem to appear
- Clip to reflector

Reflector

Object

Virtual object

Start with scene

Create virtual object

Clip to reflector
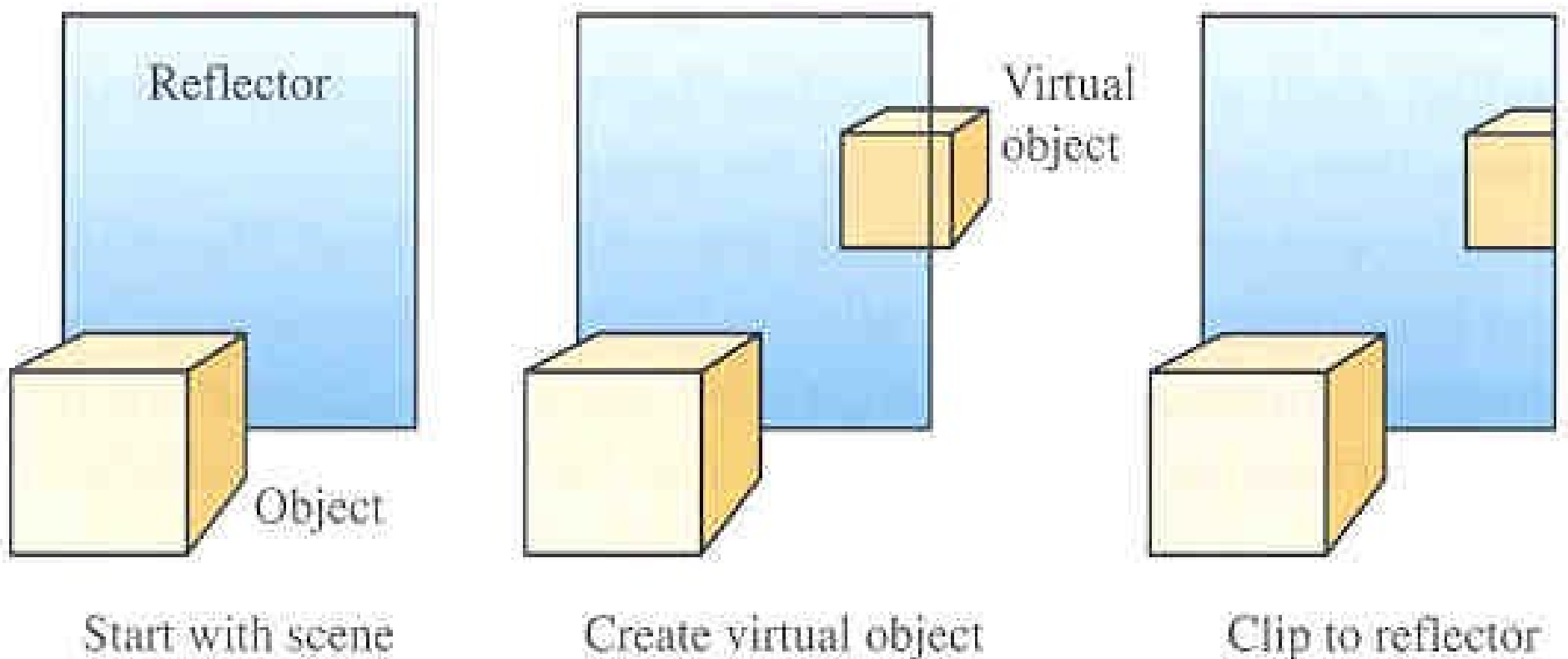
Planar Reflection Equation

- Point on mirror P
- Normal vector V

$$R = \begin{pmatrix} 1 - 2V_x^2 & -2V_xV_y & -2V_xV_z & 2(P \cdot V)V_x \\ -2V_xV_y & 1 - 2V_y^2 & -2V_yV_z & 2(P \cdot V)V_y \\ -2V_xV_z & -2V_yV_z & 1 - 2V_z^2 & 2(P \cdot V)V_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rendering Order

- Reflections are difficult when the mirror is an object inside the scene
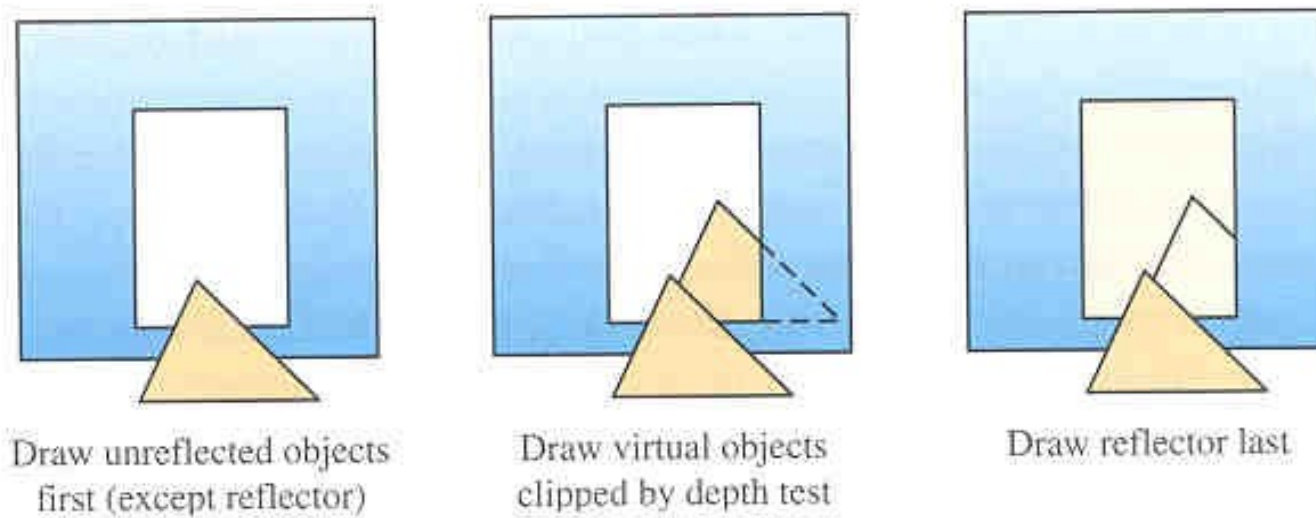 - Mirror on wall is easier

Draw unreflected objects first (except reflector)

Draw virtual objects clipped by depth test

Draw reflector last

Figure 17.3 Masking reflections with depth buffering.

Limiting the Reflector

- User defined clipping volume
  - Front and back clipping planes
  - Frustrum
- Stencil buffer
- Special cases
  - Scissors test
  - Alpha blending

Reflections using Textures

- Quads
  - Simple mirrors
- Environment maps
  - Cube map
  - Sphere map

Figure 17.4 Masking reflections using projective texture.

Reflections from Curved Surfaces

- Cannot be done using virtual objects
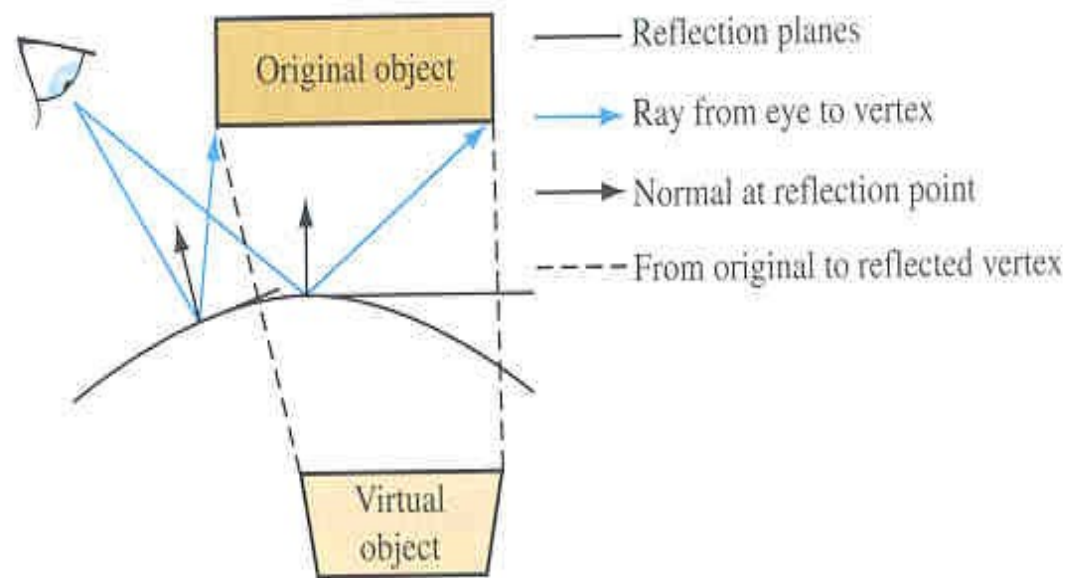- Readily done by distorting textures

Figure 17.5 Normals and reflection vectors in curved reflectors.

Inter-reflections
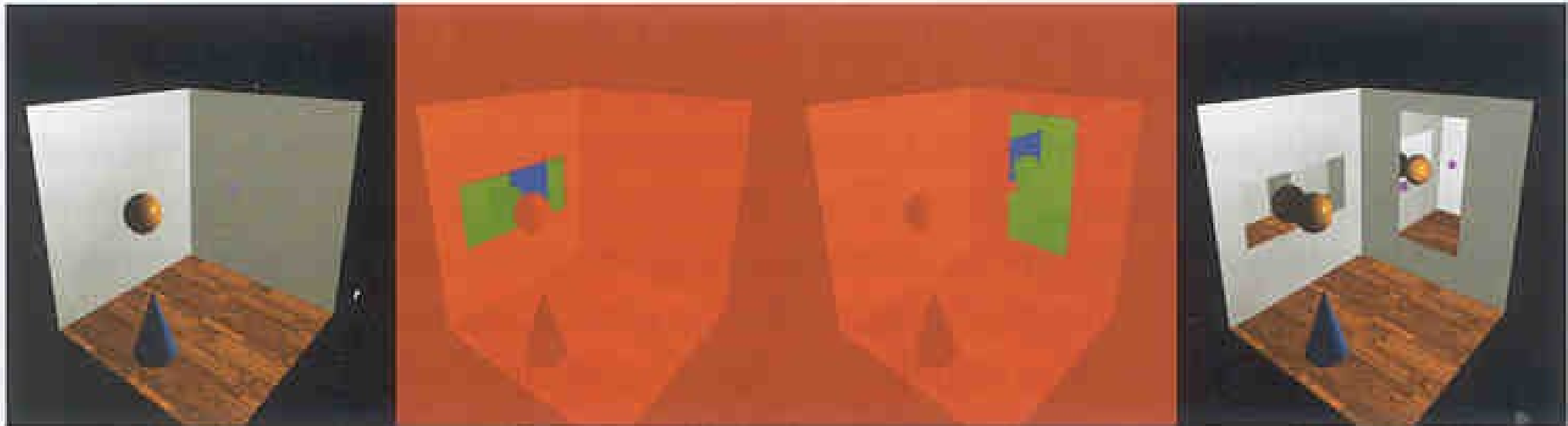
- Hall of mirrors requires multiple passes
  - Similar to max-levels

Figure 17.13 Clipping multiple interreflections with stencil.